ANALOG COMMUNICATION

Course Code :20EC07

Noise in communication and Analog Pulse modulation

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Noise

In any communication system, during the transmission of the signal, or while receiving the signal, som unwanted signal gets introduced into the communication, making it unpleasant for the receiver, questioning th quality of the communication. Such a disturbance is called as **Noise**.

Noise is an **unwanted signal** which interferes with the original message signal and corrupts the parameters of message signal. This alteration in the communication process, leads to the message getting altered. It is r likely to be entered at the channel or the receiver.

In general, noise may be predictable or unpredictable in nature. The predictable noise can be estimated eliminated by proper engineering designs.
Hence, it is understood that noise is some signal which has no pattern and no constant frequency or amplit It is quite random and unpredictable. We have no control over this noise.

Types of Noise

1. External Noise from External Source

- Atmospheric noise (due to irregularities in the atmosphere).
- Extra-terrestrial noise, such as solar noise and cosmic noise.
- Industrial noise.

Noise

2. Internal Noise from Internal Source:

This noise is produced by the receiver components while functioning. The components in the circuits, due continuous functioning, may produce few types of noise. This noise is quantifiable. A proper receiver design m lower the effect of this internal noise.

- Thermal agitation noise (Johnson noise or Electrical noise).
- Shot noise (due to the random movement of electrons and holes).
- Transit-time noise (during transition).
- Miscellaneous noise is another type of noise which includes flicker, resistance effect and mixer generated noise etc.

Effect of noise

- Degrades system performance (Analog and digital)
- Receiver cannot distinguish signal from noise
- Efficiency of communication system reduces

Noise

AM receiver:

Receiver consists of RF section, a mixer, local oscillator, an IF section , demodulator and AF power amplifier.

The incoming AM eave is received by the antenna and It is tuned and received by RF amplifier.

RF amplifier amplifying the desired RF signal and applied to mixer.

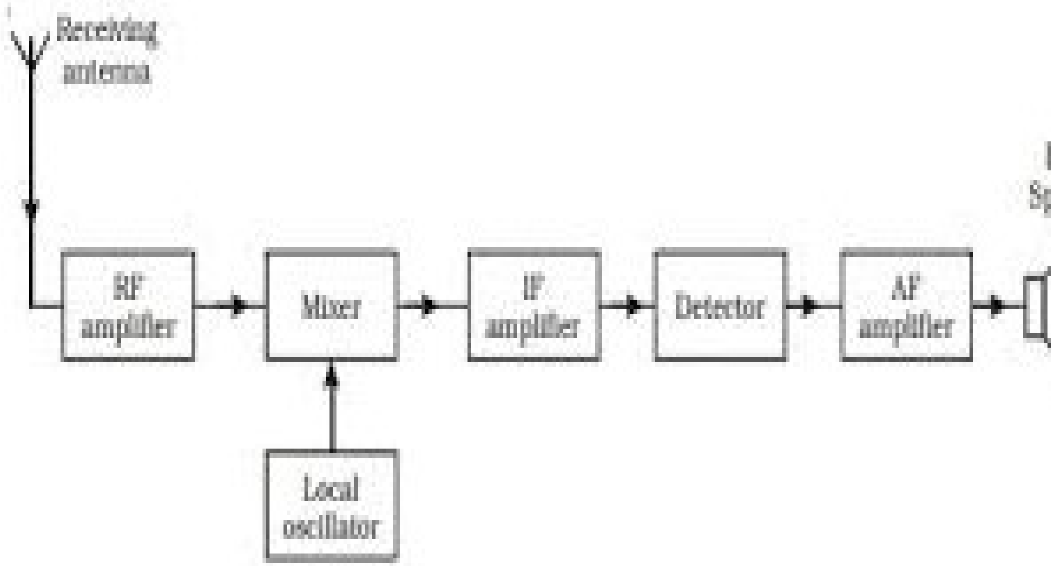
At mixer, combining or heterodyne the signal from RF amplifier and Local oscillator and provide frequency conversion, where by the incoming signal is converted to a intermediate frequency signal.

$f_{IF} = f_{LO} - f_{RF}$

Where f_{if} is intermediate frequency

f_{Lo} is local oscillator frequency

f_{rf} is frequency of the incoming RF signal



The mixer – local oscillator combination is some times referred to as the first detector and demodulator is second detector

The IF section consists of one or stages of tuned amplifiers which provides most of amplification and selectivity in the receiver.

The output of IF is applied to a demodulator to extract the original message or modulating signal, it amplified by AF ampl

Then it convert into voice signal.

Noise

Performance in Communication System:

In communication systems, message signal travels from the transmitter to the receiver via channel. The channe introduces noise and hence the message reaching the receiver becomes corrupted. As the receiver detect both noise and message signals, it reproduces a noisy message at the output.

The noise characteristics of a modulation system is evaluated by a parameter known as **Figure of Merit (Y).** It is defined as of **output Signal to Noise Ratio(SNR) to the input Signal to Noise Ratio(SNR)** of the receiver.

$$\boldsymbol{Figure\ of\ Merit}\ (Y) = \frac{(SNR)_O}{(SNR)_I}$$

The receiver performance can be defined by the **figure of merit** of the receiver.

Signal to Noise Ratio(SNR):

The common and useful measure of the fidelity of the received message is the output Signal to Noise Ratio(SNR) i defined as

$$(SNR)_O = \frac{Average\ power\ of\ message\ signal\ at\ the\ receiver\ output}{Average\ power\ of\ noise\ at\ the\ receiver\ output}$$

Noise

The output signal to noise ratio depends on the type of modulation used in the transmitter and typ
demodulation used in the receiver.

In the same way, we can define the channel Signal to Noise Ratio(SNR) is

$$(SNR)_C = \frac{Average\ power\ of\ modulated\ signal\ at\ the\ receiver\ input}{Average\ power\ of\ noise\ at\ the\ receiver\ input}$$

$$Figure\ of\ Merit\ (Y) = \frac{(SNR)_O}{(SNR)_C}$$

Clearly, higher the value of **Figure of Merit,** the better the performance of the receiver

Noise

AM receiver Model:

For the purpose of evaluating the effect of noise on the performance of a super heterodyne receiver, we consider model shown

Consider the model of an AM receiver consists of an equivalent IF filter and demodulator.
The equivalent IF filter represents the cascade filtering characteristics of the RF and IF sections of the receiver.

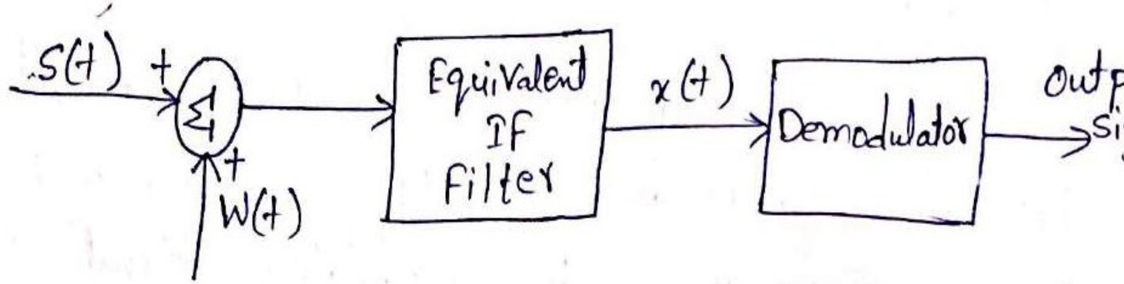


fig:- Model of an AM receiver.

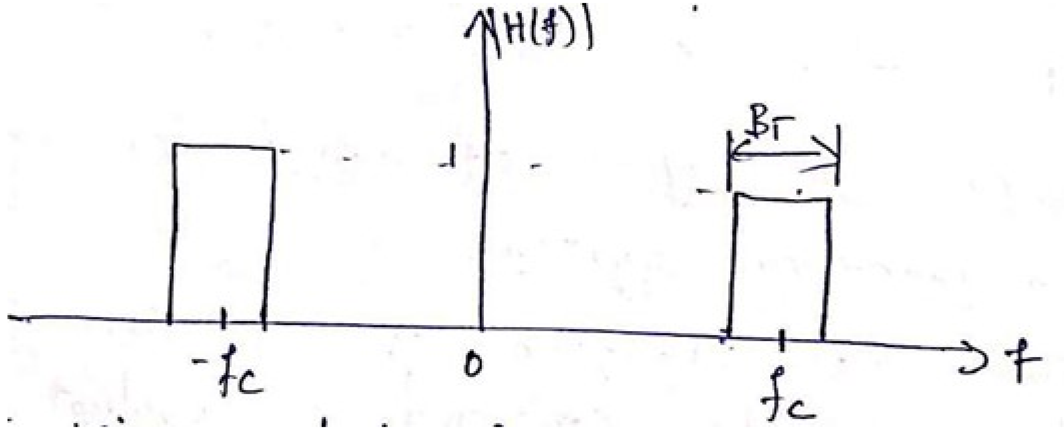
Calculation of Figure of Merit for various Communication Systems:

The noise performance of various communication systems can be compared by evaluating their Figure of Merit 'Y
For convenience, in analysis the following assumption are made:

1. The signal at filter input consists of received modulated signal s(t) and front end receiver noise.
2. The noise w(t) is modelled as white Gaussian noise of zero mean and with the power spectral density $N_0/2$.
where N0 is the average noise power per unit bandwidth measured at front end of the receiver.
3. The equivalent IF filter is usually tuned, so that its mid band frequency is same as the carrier frequency of t modulated signal s(t).i.e The bandwidth of the IF filter is just wide enough to accommodate the bandwidth modulated signal.

Noise

Therefore for convenience in SNR analysis, we assume that equivalent IF filter has Ideal band pass characteristics shown in figure.



i.e. At mixer output $f_c = f_{IF}$ and bandwidth refers to the transmission bandwidth

White noise refers to **noise that is produced by combining of all audible sound frequencies**. **Signal having equal intensity at different frequencies, giving it a constant** power spectral density.

4). The signal x(t), at the IF filter output is given by

$$x(t) = s(t) + n(t)$$

Where n(t) is a band limited white noise with power spectral density

$$S_N(f) = \frac{N_0}{2} \ , \ f_c - \frac{B_T}{2} \le f \le f_c + \frac{B_T}{2}$$

$$S_N(f) = 0, otherwise$$

Noise

5. The average noise power at the input can be calculated as

Noise Power = Power spectral density of noise X Band width

For evaluating figure of merit, the input noise power

$$N_I = \frac{N_0}{2} X\, 2W = N_0\, W$$

This will be taken as reference input noise power for evaluating figure of merit (Y) of various AM and FM receivers.

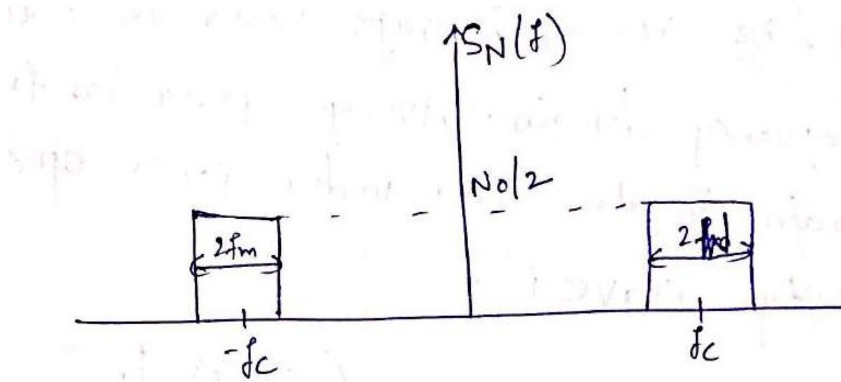


Fig:. Power density spectrum of ban pass white noise

Average power of a signal can be calculated either in time domain or in frequency domain. If signal is represent time domain, we calculate the average power in time domain.

$$Average\ Power = \frac{1}{2\pi}\int_0^{2\pi} x^2(t)dt$$

When signal is random or unknown signal like noise, average power is calculated in frequency domain. The avera power in frequency domain is the area under power spectral density curve i.e.

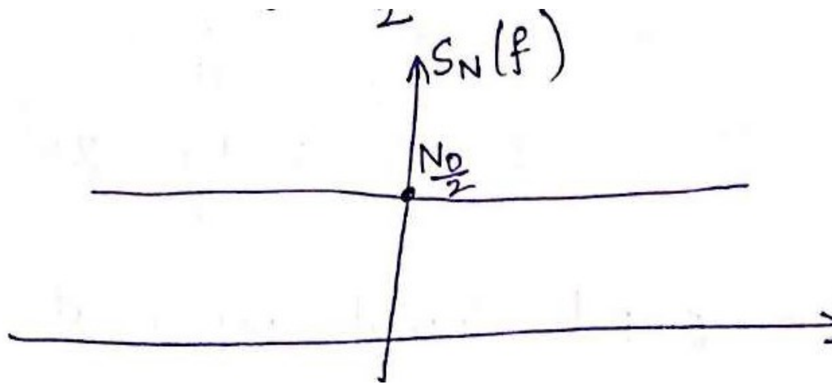
$$Average\ Power = \int_{-\infty}^{\infty} S_N(f)\, df$$

Where $S_N(f)$ is the power spectral density of noise

Noise

For a white noise, the power spectral density is defined as

$$S_N(f) = \frac{N_0}{2} \text{ and it can be represented as}$$



This wide band noise is passed through a narrow band pass filter, resulting in narrow band noise n(t).

Any band pass signal can be represented in terms of its in phase components and quadrature phase component.
For example the narrow band noise n(t) at the output of IF section can be represented as

$$n(t) = n_c(t)\cos(2\pi f_c t) - n_S(t)\sin(2\pi f_c t)$$

Where $n_c(t)$ is inphase component of n(t) and $n_s(t)$ is quadrature phase component of n(t)

Noise

Steps for calculating figure of merit (Y):

1. Calculate the average power at the input for a given modulated signal.
2. Calculate the input noise power by assuming the noise as white noise. The input noise power is always calcu for a bandwidth of '2W'.

$$Average\ input\ noise\ Power(N_I) = \int_{-\infty}^{\infty} S_N(f)\, df = \int_{-W}^{W} \frac{N_0}{2} df = \frac{N_0}{2}(2W) = N_0\, W$$

3. Calculate the output signal power at the output of demodulator.
4. The output of demodulator contains signal and narrow band noise, then the signal power is calculated from signal component and noise power is calculated from noise component.
5. Finally obtain the figure of merit as a ratio of output SNR to Input SNR.

Noise

Noise in DSB-SC System using coherent detection:

In DSBSC, the modulated signal is given as

$$s(t) = A_c \cos(2\pi f_c t)\, m(t)$$

Average signal power is given at input side of receiver

$$S_I = \frac{1}{2\pi}\int_0^{2\pi} x^2(t)dt = \frac{1}{2\pi}\int_0^{2\pi} A_c^2 \cos^2(2\pi f_c t)\, m^2(t)dt$$

$$S_I = \frac{A_c^2}{2\pi}\int_0^{2\pi} m^2(t)\left(\frac{1+\cos 4\pi f_c t}{2}\right)dt$$

$$= \frac{A_c^2}{2}\left[\frac{1}{2\pi}\int_0^{2\pi} m^2(t)dt + \frac{1}{2\pi}\int_0^{2\pi}\cos(4\pi f_c t)m^2(t)dt\right]$$

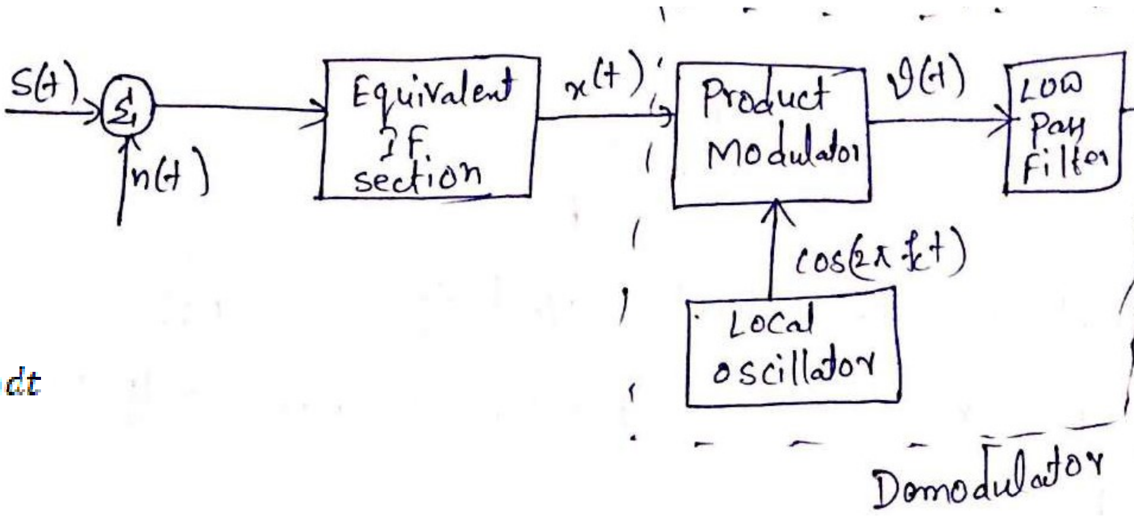


Fig. Model of DSB-SC receiver using coherent dete

$$p = \frac{A_c^2}{2}\left[\frac{1}{2\pi}\int_0^{2\pi} m^2(t)dt\right.$$

Second term is zero,

Integral cos is sin, sin 0 or π =0

$$S_I = \frac{A_c^2}{2}(P)$$

Where ‘p’ is the power of message signal.

And Input noise power is

$$N_I = \int_{-w}^{w} S_N(f)df = \int_{-w}^{w}\frac{N_0}{2}df = \frac{N_0}{2}(2W) = N_0 W$$

$$(SNR)_c = \frac{A_c^2 P}{2N_0 W}$$

Noise

Calculation of output Signal to Noise Ratio(SNR)$_O$:

Referring to the figure, we have output of IF section is

$x(t) = s(t) + n(t)$ where n(t) is the narrow band noise.

i.e. $n(t) = n_c(t)\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t)$

Therefore $V(t) = x(t)\cos(2\pi f_c t) = [\, n_c(t)\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t) + A_C\cos(2\pi f_c t)\, m(t)\,]\cos(2\pi f_c t)$

$V(t) = [\, n_c(t)\cos^2(2\pi f_c t) - n_s(t)\sin(2\pi f_c t)\cos(2\pi f_c t) + A_C\cos^2(2\pi f_c t)\, m(t)$

$$V(t) = n_c(t)\left[\frac{1+\cos(4\pi f_c t)}{2}\right] - n_s(t)\left[\frac{\sin(4\pi f_c t)}{2}\right] + A_c m(t)\left[\frac{1+\cos(4\pi f_c t)}{2}\right]$$

$$V(t) = \frac{n_c(t)}{2} + \frac{A_c m(t)}{2} + \frac{n_c(t)}{2}\cos(4\pi f_c t) - \frac{n_s(t)}{2}\sin(4\pi f_c t) + \frac{A_c m(t)}{2}\cos(4\pi f_c t)$$

When the signal is passed through a low pass filter, reject high frequency component and allow low frequency components.

$$y(t) = \frac{n_c(t)}{2} + \frac{A_c m(t)}{2}$$

Therefore the output of demodulator consists of message m(t) and inphase noise component $n_c(t)$ appea additively at the receiver output.

Noise

All other components are removed by low pass filter, since they contains $4\pi f_c t$ term.

The output of signal power is given by

$$S_0 = \frac{1}{2\pi}\int_0^{2\pi} y^2(t)dt = \frac{1}{2\pi}\int_0^{2\pi} \frac{A_c{}^2}{4} m^2(t)dt = \frac{A_c{}^2}{4}\frac{1}{2\pi}\int_0^{2\pi} m^2(t)dt = \frac{A_c{}^2}{4}\ P$$

And output noise power is given by

$$N_{0P} = \frac{1}{2\pi}\int_0^{2\pi} \frac{n_c^2}{4} dt = \frac{1}{4}\frac{1}{2\pi}\int_0^{2\pi} n_c^2\ dt$$ Time domain

$$= \frac{1}{4}\int_{-\infty}^{\infty} S_{N_c}(f)\quad df$$ frequency domain

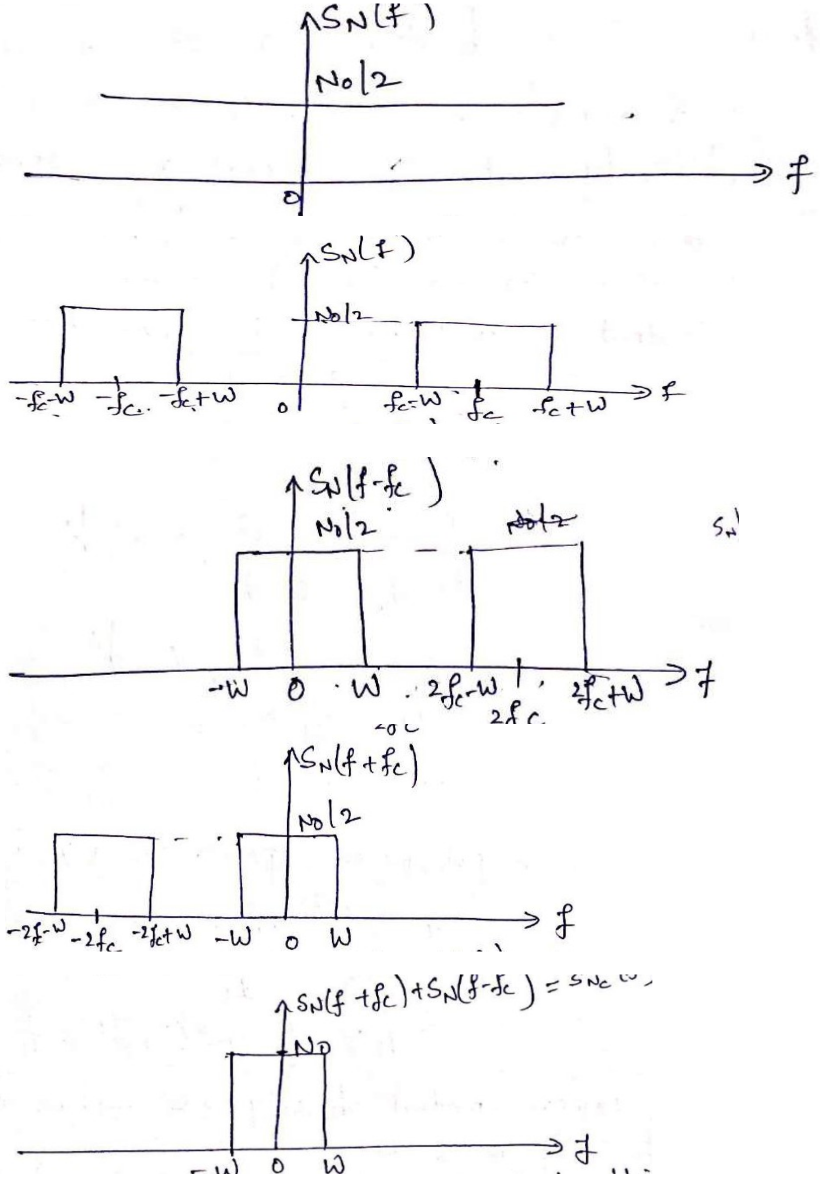
The power spectral density of inphase component can be represented as

$$S_{N_c}(f) = S_N(f - f_c) + S_N(f + f_c)$$

Where $S_{N_c}(f)$ is the power spectral density of inphase noise component.

$S_N(f)$ is the power spectral density of noise

Noise



Output noise power is

$$N_{OP} = \frac{1}{4}\int_{-W}^{W} N_o\, df = \frac{N_o}{4}[2W] = \frac{N_oW}{2}$$

$$(SNR)_O = \frac{A_c^2P}{2N_oW}$$

Figure of merit for DSBSC is

$$Figure\ of\ merit(\gamma) = \frac{SNR_o}{SNR_c} = \frac{\frac{A_c^2P}{2N_oW}}{\frac{A_c^2P}{2N_oW}}$$

$$\boldsymbol{Figure\ of\ merit(\gamma) = 1\ for\ DSBSC}$$

Noise in DSBFC or AM receiver using Envelope detector:

In a full amplitude modulated wave i.e. AM wave, both side bands and carrier are transmitted is given by

$$s(t) = A_c[1 + k_a m(t)]\cos(2\pi f_c t)$$

Below figure shows to evaluate the noise performance of an AM receiver.

The input side signal is AM signal and the average power of signal at input side is calculated by

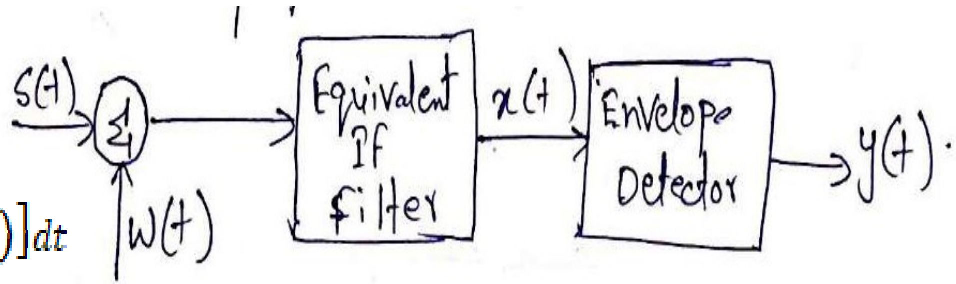


Fig.: Model of AM receiver using Envelope de

$$S_I = \frac{1}{2\pi}\int_0^{2\pi} s^2(t)dt = \frac{1}{2\pi}\int_0^{2\pi}[A_c^2\cos^2(2\pi f_c t) + A_c^2 k_a^2 m^2(t)\cos^2(2\pi f_c t)]dt$$

$$= \frac{A_c^2}{2\pi}\left[\int_0^{2\pi}\left[\left(\frac{1+\cos 4\pi f_c t}{2}\right) + k_a^2 m^2(t)\left(\frac{1+\cos 4\pi f_c t}{2}\right)\right]dt\right]$$

Third term i.e 2ab neglected, i
integration of cos π is sin π=0

$$= \frac{A_c^2}{2\pi}\left[\frac{1}{2}(2\pi - 0) + 0\right] + \frac{A_c^2}{2\pi}\left[k_a^2\int_0^{2\pi}\frac{m^2(t)}{2} + 0\right]$$

$$S_i = \frac{A_c^2}{2} + k_a^2\frac{A_c^2}{2}.P$$

Where ‘P’ is the power of message given by $P = \frac{1}{2\pi}\int_0^{2\pi} m^2(t)\,dt$

$$S_i = \frac{A_c^2}{2}[1 + k_a^2 P]$$

Noise

$$Average\ input\ noise\ Power N_I = \int_{-w}^{w} S_N(f)\,df = \int_{-w}^{w} \frac{N_0}{2} df = \frac{N_0}{2}(2W) = N_0\,W$$

Therefore signal to noise ratio at input side of the receiver $(SNR)_c$ is given by

$$(SNR)_c = \frac{average\ power\ of\ input\ signal}{average\ power\ of\ noise\ at\ input\ of\ receiver}$$

$$= \frac{\frac{A_c^2}{2}[1 + k_a^2 P]}{N_0\,W} = \frac{A_c^2[1 + k_a^2 P]}{2N_0\,W} \quad \text{-----------(1)}$$

Referring to the block diagram of AM receiver model, **x(t) = s(t) + n(t)**

$$x(t) = [A_c + A_c k_a m(t)]\cos(2\pi f_c t) + n_c(t)\cos(2\pi f_c t) - n_S(t)\sin(2\pi f_c t)$$

$$x(t) = [A_c + A_c k_a m(t) + n_c(t)]\cos(2\pi f_c t) - n_S(t)\sin(2\pi f_c t) \quad \text{-----------(2)}$$

This signal x(t) which is given as input to the envelope detector. Then the output of envelope detector y(t) is given as envelope i.e. Magnitude of signal x(t).

$$y(t) = \sqrt{(A_c + A_c k_a m(t) + n_c(t))^2 + (n_S^2(t)}$$

Noise

This signal y(t) can also be obtained by representing the components of equation(2) by mean of phasors as shown in fig.

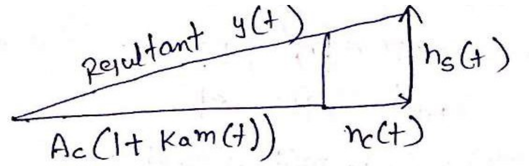


fig:- phasor diagram of AM wave Narrowband Nois

The resultant of phasor is the receiver output obtained as

y(t) = envelope of x(t)

$$y(t) = \sqrt{(A_c + A_c k_a m(t) + n_c(t))^2 + (n_s^2(t)}$$

This output y(t) is the output of an ideal envelope detector. To make the noise analysis simple, assuming tha signal power is large compared to the average noise power, then the signal term $A_c + A_c k_a m(t)$ will large comp with noise term $n_c(t)$ + $n_s(t)$ and the quadratic component $n_s(t)$ can be neglected. The above equation ca approximated as follows

$$y(t) \simeq [A_c + A_c k_a m(t) + n_c(t)]$$

The above expression contains one inphase noise component $n_c(t)$, one message component and one car component. The d.c.or constant term A_c removed simply by means of blocking capacitor. Thus if we neglect the t A_C in the above expression, then y(t) i.e output of demodulator can be given as

$$y(t) = A_c k_a m(t) + n_c(t)$$

Radio Receivers

The average signal power at the output of receiver can be given by

$$S_o = \frac{1}{2\pi}\int_0^{2\pi} s^2(t)dt = \frac{1}{2\pi}\int_0^{2\pi} A_c^{\,2} k_a^{\,2} m^2(t)dt = A_c^{\,2} k_a^{\,2} \frac{1}{2\pi}\int_0^{2\pi} m^2(t)\,dt = A_c^{\,2} k_a^{\,2} P$$

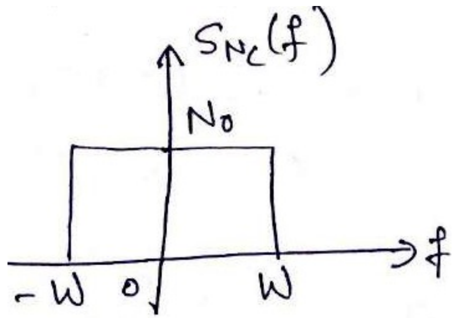
The average noise power of the output of receiver can be give by

$$N_{op} = \frac{1}{2\pi}\int_0^{2\pi} n_c^{\,2}(t)dt = \int_{-\infty}^{\infty} S_{N_c}(f)df$$

Where $S_{N_c}(f)$ is the power spectral density of inphase noise component given by

$$S_{N_c}(f) = S_N(f - f_c) + S_N(f + f_c)$$

And $S_{N_c}(f)$ hsa no over the range between –W to W



The output noise power is $N_{op} = \int_{-W}^{W} N_o\,df = N_o(2W) = 2N_oW$

The SNR at output of the receiver is

$$(SNR)_o = \frac{average\ power\ of\ input\ signal}{average\ power\ of\ noise\ at\ input\ of\ receiver} = \frac{A_c^{\,2} k_a^{\,2} P}{2N_oW}$$

Noise

The figure of merit(Y) of a AM receiver is given by

$$Figure\ of\ merit(\gamma) = \frac{SNR_o}{SNR_c} = \frac{\frac{A_c{}^2 k_a{}^2 P}{2N_o W}}{\frac{A_c^2\left[1 + k_a^2 P\right]}{2N_0 W}} = \frac{k_a{}^2 P}{1 + k_a^2 P}$$

The figure of merit(Y) of a AM or DSBSC receiver is always less than unity.

Noise

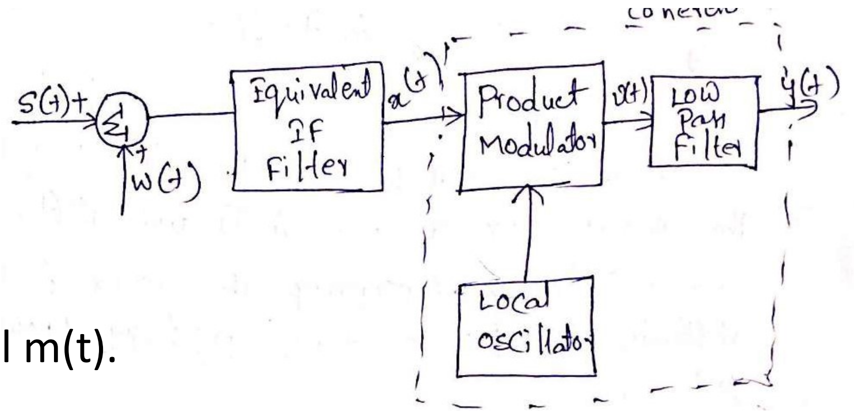
Signal to Noise Ratio in SSB receiver using coherent detection:

The model of SSB receiver using coherent detector is shown in figure

We assume that only lower sideband is transmitted, so that the modulated wave is expressed as

$$s(t) = \frac{A_c}{2}\cos(2\pi f_c t)m(t) + \frac{A_c}{2}\hat{m}(t)\sin(2\pi f_c t)$$

Where $\hat{m}(t)$ is the Hilbert transform of message signal m(t).



coherent detector

Average power of signal at input side of receiver is given by

$$S_I = \frac{1}{2\pi}\int_0^{2\pi} s^2(t)dt = \frac{1}{2\pi}\int_0^{2\pi}\frac{A_c^2}{4}\cos^2(2\pi f_c t)\, m^2(t)dt + \frac{1}{2\pi}\int_0^{2\pi}\frac{A_c^2}{4}\sin^2(2\pi f_c t)\,\hat{m}^2(t)dt$$

$$S_I = \frac{A_c^2}{4}\frac{1}{2\pi}\int_0^{2\pi}\left(\frac{1+\cos 4\pi f_c t}{2}\right)m^2(t)dt + \frac{1}{2\pi}\int_0^{2\pi}\frac{A_c^2}{4}\left(\frac{1-\cos 4\pi f_c t}{2}\right)\hat{m}^2(t)dt$$

$$S_I = \frac{A_c^2}{4}\frac{1}{2\pi}\int_0^{2\pi}\frac{m^2(t)}{2}dt + \frac{A_c^2}{4}\frac{1}{2\pi}\int_0^{2\pi}\frac{m^2(t)\cos 4\pi f_c t}{2}dt$$

$$+\frac{A_c^2}{4}\frac{1}{2\pi}\int_0^{2\pi}\frac{\hat{m}^2(t)}{2}dt - \frac{A_c^2}{4}\frac{1}{2\pi}\int_0^{2\pi}\frac{\hat{m}^2(t)\cos 4\pi f_c t}{2}dt$$

Noise

$$S_I = \frac{A_c^{\,2}}{4}\frac{1}{2\pi}\int_0^{2\pi}\frac{m^2(t)}{2}dt + o + \frac{A_c^{\,2}}{4}\frac{1}{2\pi}\int_0^{2\pi}\frac{\hat{m}^2(t)}{2}dt - o$$

$$S_I = \frac{A_c^{\,2}}{8}P + \frac{A_c^{\,2}}{4}\frac{1}{2\pi}\int_0^{2\pi}\frac{\hat{m}^2(t)}{2}dt$$

In this expression, $\hat{m}(t)$ is the Hilbert transform of a message signal m(t). Therefore, the average power of m(t) a are same. Therefore considering the power of signal $\hat{m}(t)$ is also P. Therefore, input signal is given by

$$S_I = \frac{A_c^{\,2}}{8}P + \frac{A_c^{\,2}}{8}P = 2\frac{A_c^{\,2}}{8}P = \frac{A_c^{\,2}}{4}P$$

Average noise power at the input side of receiver is

$$Average\ input\ noise\ Power N_I = \int_{-w}^{w} S_N(f)\,df = \int_{-w}^{w}\frac{N_0}{2}df = \frac{N_0}{2}(2W) = N_0\,W$$

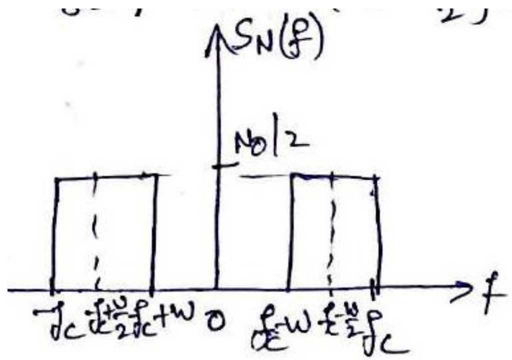
Power spectral density of noise component is given by $S_N(f) = \frac{N_0}{2}$

Signal to Noise ratio at input side of receiver is

$$(SNR)_c = \frac{average\ power\ of\ input\ signal\ of\ receiver}{average\ power\ of\ noise\ at\ input\ of\ receiver} = \frac{\frac{A_c^{\,2}}{4}P}{N_0W} \qquad (SNR)_c = \frac{A_c^2P}{4N_0W}$$

Radio Receivers

Referring block diagram, the output of the equivalent IF filter is



$$X(t) = s(t) + n(t)$$

$$s(t) = \frac{A_c}{2}\cos(2\pi f_c t)m(t) + \frac{A_c}{2}\hat{m}(t)\sin(2\pi f_c t)$$

$$n(t) = n_c(t)\cos 2\pi(f_c - \frac{W}{2})t \quad - n_s(t)\sin 2\pi(f_c - \frac{W}{2})t$$

$$V(t) = x(t)\cos 2\pi f_c t$$

$$V(t) = (s(t) + n(t))\cos 2\pi f_c t$$

$$V(t) = \left[\frac{A_c}{2}\cos(2\pi f_c t)m(t) + \frac{A_c}{2}\hat{m}(t)\sin(2\pi f_c t)\right]\cos(2\pi f_c t) + \left[n_c(t)\cos 2\pi(f_c - \frac{W}{2})t \quad - n_s(t)\sin 2\pi(f_c - \frac{W}{2})t\right]\cos(2\pi f_c t)$$

$$V(t) = \left[\frac{A_c}{2}\cos^2(2\pi f_c t)m(t) + \frac{A_c}{2}\hat{m}(t)\frac{\sin 4\pi f_c t}{2}\right] + \left[n_c(t)\cos(2\pi f_c t)\cos 2\pi(f_c - \frac{W}{2})t \quad - n_s(t)\cos(2\pi f_c t)\sin 2\pi(f_c - \frac{W}{2})t\right]$$

Radio Receivers

$$V(t) = \left[\frac{A_c}{2} m(t)\left(\frac{1 + cos4\pi f_c t}{2}\right) + \frac{A_c}{2}\hat{m}(t)\frac{sin4\pi f_c t}{2}\right] + \left[\frac{n_c(t)}{2}[\cos(4\pi f_c t - \pi wt) + \cos(\pi wt)] - \frac{n_s(t)}{2}[\sin(4\pi f_c t - \pi wt) - \sin(\pi wt)]\right]$$

After passing through the low pass filter

$$y(t) = \frac{A_c}{4} m(t) + \frac{n_c(t)}{2}\cos(\pi wt) + \frac{n_s(t)}{2}\sin(\pi wt)$$

The average signal power at output of receiver is

$$S_o = \frac{1}{2\pi}\int_0^{2\pi} s^2(t)dt = \frac{1}{2\pi}\int_0^{2\pi}\frac{A_c{}^2}{16} m^2(t)dt = \frac{A_c{}^2}{16}\frac{1}{2\pi}\int_0^{2\pi} m^2(t)\,dt = \frac{A_c{}^2}{16} P$$

$$P = \frac{1}{2\pi}\int_0^{2\pi} m^2(t)\,dt$$

The output noise power can be given as

$$N_{OP} = \frac{1}{4}\frac{1}{2\pi}\int_0^{2\pi}[n_c(t)\cos(\pi wt)]^2\,dt + \frac{1}{4}\frac{1}{2\pi}\int_0^{2\pi}[n_s(t)\sin(\pi wt)]^2\,dt$$

$$N_{OP} = \frac{1}{4}\frac{1}{2\pi}\int_0^{2\pi}[n_c^I(t)]^2\,dt \mid \frac{1}{4}\frac{1}{2\pi}\int_0^{2\pi}[n_s^I(t)]^2\,dt$$

Radio Receivers

$$N_{OP} = \frac{1}{4}\frac{1}{2\pi}\int_0^{2\pi} n_c^2\,(t)\left(\frac{1 + cos2\pi wt}{2}\right) dt + \frac{1}{4}\frac{1}{2\pi}\int_0^{2\pi} n_s^2(t)\left(\frac{1 - cos\pi wt}{2}\right) dt$$

$$N_{OP} = \frac{1}{4}\frac{1}{2\pi}\int_0^{2\pi} \frac{n_c^2(t)}{2} dt + \frac{1}{4}\frac{1}{2\pi}\int_0^{2\pi} \frac{n_s^2(t)}{2} dt$$

$$N_{OP} = \frac{1}{8}\frac{1}{2\pi}\int_0^{2\pi} n_c^2(t)\,dt + \frac{1}{8}\frac{1}{2\pi}\int_0^{2\pi} n_s^2(t)\,dt = \frac{1}{8}\int_{-\infty}^{\infty} S_{N_c}(f)df + \frac{1}{8}\int_{\infty}^{-\infty} S_{N_c}(f)df$$

Where $S_{Nc}(f)$ and $S_{Ns}(f)$ are the power spectral densities of inphase and quadrature phase noise components i.e. and $n_s(t)$.

$$S_{N_c}(f) = S_{N_s}(f) = S_N\left[f - \left(f_c - \frac{W}{2}\right)\right] + S_N\left[f + \left(f_c - \frac{W}{2}\right)\right]$$

This is because, the center frequency of noise component of SSB is $f_c - \frac{W}{2}$. This can be given by $S_{Nc}(f) = S_{Ns}(f) =$ Over the limits –W/2 to W/2.

The output noise power is given by

$$N_{OP} = \frac{1}{8}\int_{-w/2}^{w/2} N_o\,df + \frac{1}{8}\int_{-w/2}^{w/2} N_o\,df = \frac{1}{8}N_o(W) + \frac{1}{8}N_o(W) = \frac{N_o(W)}{4}$$

Radio Receivers

Output signal to noise ratio is given by

$$(SNR)_o = \frac{average\ power\ of\ output\ signal\ at\ receiver}{average\ power\ of\ noise\ at\ output\ at\ receiver} = \frac{\frac{{A_c}^2}{16}P}{\frac{N_o(W)}{4}} = \frac{{A_c}^2 P}{4N_o W}$$

$$Figure\ of\ Merit(\gamma) = \frac{(SNR)_o}{(SNR)_c} = \frac{\frac{{A_c}^2 P}{4N_o W}}{\frac{A_c^2 P}{4N_0 W}} = 1$$

Radio Receivers

Threshold effect on AM Receiver: The loss of message m(t) in an envelope detector due to the presence of large noise is referred as '**Threshold effect**'.

The output of an envelope detector of AM receiver is given by

$$y(t) = \sqrt{(A_c + A_c k_a m(t) + n_c(t))^2 + (n_s^2(t)}$$

In this case consider, the noise power is large compared to the signal power

$$y(t) = \sqrt{[(A_c + m(t)) + n_c(t)]^2 + n_s^2 t}$$

$$y(t) = \sqrt{[(A_c + m(t)]^2 + n_c^2(t) + 2(A_c + m(t))n_c(t) + n_s^2 t}$$

$$y(t) = \sqrt{n_c^2(t) + n_s^2 t + 2n_c(t)(A_c + m(t))}$$

This is because neglecting $[(A_c + m(t)]^2$, Since we had considered signal power is small i.e. Ac +m(t) is small, $[(A_c + m(t)]^2$ will be very small and can be neglected.

Radio Receivers

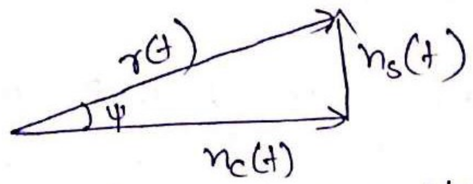
$$y(t) = \sqrt{(n_c^2(t) + n_s^2(t))\left[1 + \frac{2n_c(t)(A_c + m(t))}{(n_c^2(t) + n_s^2(t)}\right]}$$

$$Consider\ r(t) = \sqrt{n_c^2(t) + n_s^2(t)}$$

$$\varphi(t) = \tan^{-1}\frac{n_s(t)}{n_c(t)}$$

Consider the phasor diagram of noise components as given below

$$y(t) = \sqrt{r^2(t)\left[1 + \frac{2(A_c + m(t))}{r(t)} X \frac{n_c(t)}{r(t)}\right]} - -(1)$$



Above equation –(1) becomes

$$y(t) = r(t)\left[1 + \frac{2(A_c + m(t))}{r(t)} cos\varphi(t)\right]^{\frac{1}{2}}$$

From the above fig we can write

$$\cos\varphi(t) = \frac{n_c(t)}{r(t)}$$

Radio Receivers

Since noise component r(t) >> 2(Ac+m(t)), then we can approximate the above equation as

$$y(t) = r(t)\left[1 + \frac{1}{2} X \frac{2(A_c + m(t))}{r(t)} cos\varphi(t)\right]$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3$$

$$y(t) = r(t)\left[1 + \frac{(A_c + m(t))}{r(t)} cos\varphi(t)\right]$$

$$\sqrt{1+x} = 1 + \tfrac{1}{2}x - \tfrac{1}{8}x^2 + \tfrac{1}{16}x^3 - \tfrac{5}{128}x^4 + \tfrac{7}{256}x^5 - \cdots$$

$$y(t) = r(t) + (A_c + m(t))cos\varphi(t)$$

$$y(t) = r(t) + A_c cos\varphi(t) + m(t)cos\varphi(t)$$

$$A_c K_a = 1$$

$$y(t) = r(t) + A_c cos\varphi(t) + A_c K_a m(t)cos\varphi(t)$$

From the above equation it is evident that, the envelope y(t) which appears at the output of the envelope detector h no component strictly proportional to message signal m(t).

The last term of above equation containing m(t) is multiplied by large noise component i.e. cosΨt. And hence m(t) cannot be separated from the noise.

As the modulating term m(t) is completely mixing with noise, it carries no useful information.

Radio Receivers

The loss of message m(t) in an envelope detector due to the presence of large noise is referred as '**Threshold ef**
When the noise is large compared to the signal at the input of envelope detector, the detected output has a me
signal completely mingled with noise.

This means that, if the input SNR is below a certain level called 'threshold level' noise dominates the message si

Threshold is nothing but, minimum signal to noise ratio.

Threshold is defined as a value of input SNR below which the output SNR deteriorates much more rapidly than i
SNR.

Radio Receivers

FM Receivers:

The frequency range of commercial FM radios : 88-108 MHZ
Mid band frequency of IF amplifier: 10.7MHz
IF bandwidth: 0.2MHZ

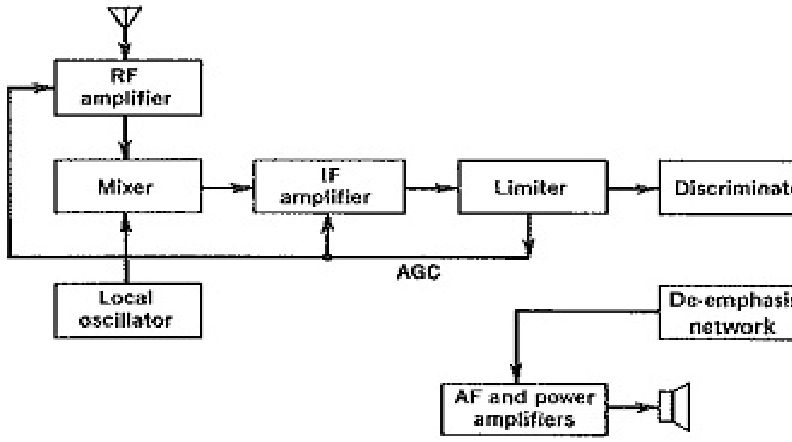


FIGURE 6-28 FM receiver block diagram.

FM Receiver Model:

For the purpose of evaluating the performance of an FM receiver in the presence of noise w(t), we may use th model shown in below figure.

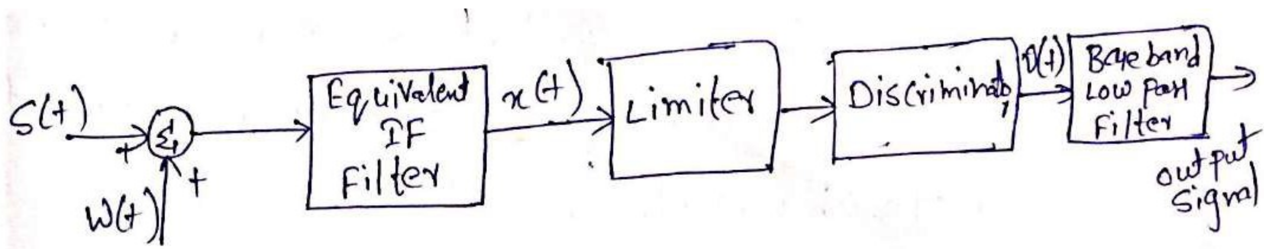
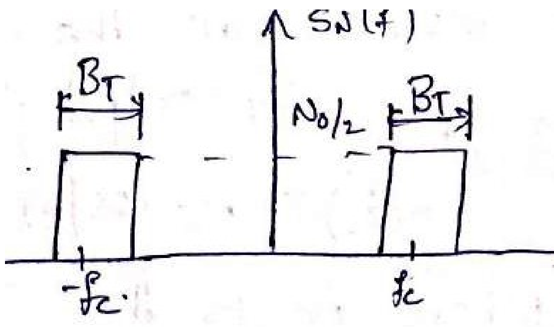


fig a, Model of an FM Receiver.

Radio Receivers

W(t) is modelled as white Gaussian noise of zero mean and power spectral density $N_0/2$.
The equivalent IF filter represents combined filtering characteristics of RF and IF sections of the receiver. This has a mid-band frequency f_c and bandwidth B_T.
We assume that IF filter has an ideal band pass characteristics o with bandwidth B_T small compared with the mid frequency fc. Thus may be use the usual narrowband representation for the filtered noise n(t) in terms of inphase quadrature phase components.



The limiter removes any amplitude variations at the IF output.
The discriminator is assumed to be ideal in the sense that its output is proportional to the deviation in the instantaneous frequency of the carrier away from f_c.
Also the post detection filter is assumed to be an ideal low pass filter with a bandwidth equal to the message bandwidth 'W'.

Noise in FM receiver:

The narrow band noise n(t) at the IF filter output is defined in terms of its in-phase and quadrature phase compon by

i.e. $n(t) = n_c(t)\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t)$ -----(1)

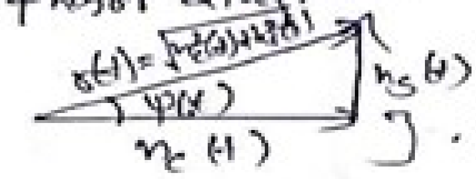
Radio Receivers

Equivalently we may express n(t) in terms of its envelope and phase as

$$n(t) = r(t)\cos(2\pi f_c t + \varphi(t))$$

where $r(t) = \sqrt{n_c^2(t) + n_s^2(t)}$ $\qquad \varphi(t) = \tan^{-1}\dfrac{n_s(t)}{n_c(t)}$

[∵ Multiply and divide eq ① with $\sqrt{n_c^2(t)+n_s^2(t)}$ and use the below phasor diagram



].

We assume that the FM signal at the IF Filter output is

$$s(t)=A_c\cos\left(2\pi f_c t+2\pi k_f\int m(t)dt\right)$$

A_c is carrier amplitude, f_c is carrier frequency, K_f is frequency sensitivity, m(t) is message or modulating wave

For convenience , we define

$$\emptyset(t) = 2\pi k_f \int_0^t m(t)dt$$

$$s(t) = A_c Cos(2\pi f_c t + \emptyset(t))$$

Average Input signal power is given by $S_i = \dfrac{A_c^2}{2}$

Signal to Noise Ratio at the input of receiv

$$(SNR)_c = \frac{A_c^2}{2N_oW}$$

Average Input noise power is given by $N_i = N_oW$

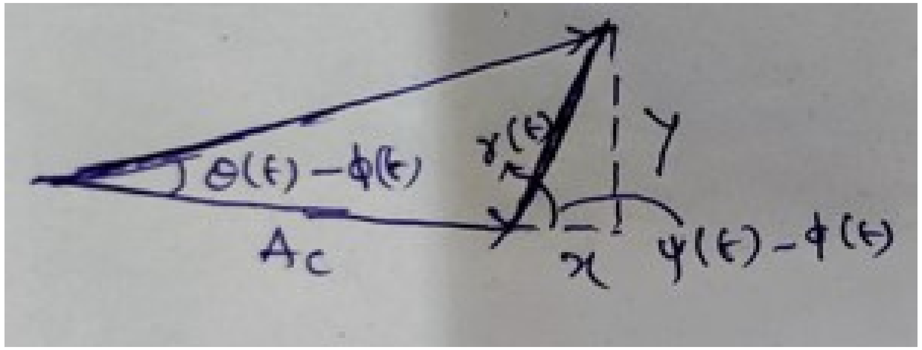
Radio Receivers

Therefore the total signal at IF section output is

X(t) = S(t) + n(t)

$$x(t) = A_c cos[2\pi f_c t + \emptyset(t)] + r(t)cos[2\pi f_c t + \varphi(t)]$$

Let Ø(t) be the relative phase angle which varies as a function of message signal, then a phasor diagram is draw by assuming large value of carrier to noise ratio and taking the carrier component as reference.



$$cos\ [\varphi(t) - \emptyset(t)] = \frac{x}{r(t)}$$

$$x = r(t)Cos\ [\varphi(t) - \emptyset(t)]$$

$$sin\ [\varphi(t) - \emptyset(t)] = \frac{y}{r(t)}$$

$$y = r(t)sin\ [\varphi(t) - \emptyset(t)]$$

$$tan\ [\theta(t) - \emptyset(t)] = \frac{y}{A_c + x} = \frac{r(t)sin\ [\varphi(t) - \emptyset(t)]}{A_c + r(t)Cos\ [\varphi(t) - \emptyset(t)]}$$

Radio Receivers

$$\theta(t) = \emptyset(t) + \tan^{-1}\left[\frac{r(t)sin\,[\varphi(t) - \emptyset(t)]}{A_c + r(t)Cos\,[\varphi(t) - \emptyset(t)]}\right]$$

As carrier power is more, the above equation can be modified as

$$\theta(t) = \emptyset(t) + \tan^{-1}\left[\frac{r(t)sin\,[\varphi(t) - \emptyset(t)]}{A_c}\right]$$

$$for\ small\ values\ of\ \theta, \tan^{-1}\theta = \theta$$

$$\theta(t) = \emptyset(t) + \left[\frac{r(t)sin\,[\varphi(t) - \emptyset(t)]}{A_c}\right]$$

The phase angle φ(t) may vary between 0 to 2π and is independent of the message signal. Then the a expression can be modified as

$$\theta(t) = \emptyset(t) + \left[\frac{r(t)sin\,[\varphi(t)]}{A_c}\right]$$

Radio Receivers

The output of slope detector is $$V(t) = \frac{1}{2\pi}\frac{d}{dt}\theta(t)$$

$$V(t) = \frac{1}{2\pi}\frac{d}{dt}\left[\emptyset(t) + \left[\frac{r(t)sin\,[\varphi(t)]}{A_c}\right]\right]$$

$$V(t) = \frac{1}{2\pi}\frac{d}{dt}\left[2\pi k_f \int_0^t m(t)dt\right] + \frac{1}{2\pi}\frac{d}{dt}\left[\frac{r(t)sin\,[\varphi(t)]}{A_c}\right]$$

$$V(t) = \frac{1}{2\pi}2\pi k_f m(t) + \frac{1}{2\pi A_c}\frac{d}{dt}r(t)sin\,[\varphi(t)]$$

$$V(t) = k_f m(t) + \frac{1}{2\pi A_c}\frac{d}{dt}r(t)sin\,[\varphi(t)]$$

Radio Receivers

We have $n(t) = n_c(t)\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t)$ --------(1)

also $$n(t) = r(t)\cos(2\pi f_c t + \varphi(t))$$

$$n(t) = r(t)\cos(2\pi f_c t)\,cos\varphi(t) - r(t)\sin(2\pi f_c t)\,sin\varphi(t) - (2)$$

By comparing equation(1) & (2) we get $n_c(t) = r(t)cos\varphi(t)\ and\ n_s(t) = \ r(t)sin\varphi($

The output of the discriminator or slope detector is given as

$$V(t) = k_f m(t) + \frac{1}{2\pi A_c}\frac{d}{dt}n_s(t)$$

The signal power at the output of the receiver is given as $S_o = \frac{1}{2\pi}\int_0^{2\pi} K_f^2\, m^2(\mathrm{t})\mathrm{dt}$

$$S_o = K_f^2 P$$

Noise power can be obtained by using time differentiation property of Fourier Transform

$$\frac{1}{2\pi A_c}\frac{d}{dt}(n_s(t)) \overset{F.T}{\longrightarrow} = \frac{1}{2\pi A_c} J2\pi f N_s(f)$$

Radio Receivers

$$N_o(f) = \frac{jf}{A_c} N_s(f)$$

Where $N_o(f)$ is the noise at Discriminator output and $N_s(f)$ is the quadrature phase noise component

Therefore average Noise Power at the output is given by

$$N_{OP} = \frac{1}{A_c^2} \int_{-\infty}^{\infty} f^2 S_{N_s}(f) df - (1)$$

Where $S_{Ns}(f)$ is power spectral density of quadrature noise component is given by

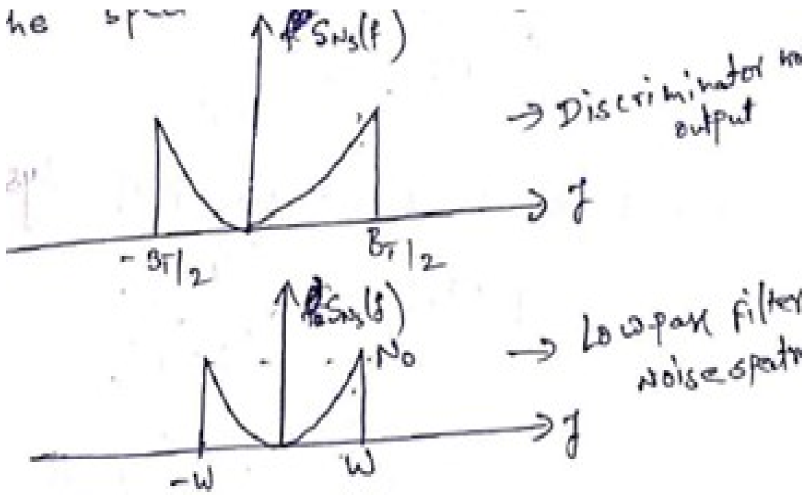
$S_{Ns}(f) = N_o$. Over the limits –W to W.

This can be proved as follows

$$S_{Ns}(f) = S_N(f - f_c) + S_N(f + f_c)$$

Radio Receivers

After passing through the low pass filter, which having maximum frequency component 'W'.
The spectrum of $f^2 S_{Ns}(f)$ is shown in figure.



$$N_{op} = \frac{1}{A_c^2}\int_{-w}^{w} f^2 N_o df = \frac{N_o}{A_c^2}\left[\frac{f^3}{3}\right]_{-w}^{w} = \frac{N_o}{3A_c^2}[w^3 - (-w^3)] = \frac{2N_o w^3}{3A_c^2}$$

Radio Receivers

Signal to Noise Ratio at the output of the receiver is given by

$$(SNR)_o = \frac{k_f^2 P}{\frac{2N_o w^3}{3A_c^2}} = \frac{3A_c^2 k_f^2 P}{2N_o w^3}$$

Average Input signal power is given by

$$S_i = \frac{A_c^2}{2}$$

Average Input noise power is given by $N_i = N_o W$

Signal to Noise Ratio at the input of receiver is

$$(SNR)_c = \frac{A_c^2}{2N_o W}$$

$$Figure\ of\ Merit = \frac{(SNR)_o}{(SNR)_c} = \frac{\frac{3A_c^2 k_f^2 P}{2N_o w^3}}{\frac{A_c^2}{2N_o W}} = \frac{3k_f^2 P}{W^2}$$

Radio Receivers

Capture Effect:

The purpose of an FM receiver is to effectively reconstruct the original message signal with mini
noise.

The presence of amplitude limiter eliminates all unwanted amplitude variations to make FM best in
performance.

In addition to this, FM receivers can also reduce interference from other stations. This is possible
when the signal from desired stations is stronger than the interfering station.

When the interference is stronger than the desired signal, then the receiver locks on to the stronger s
and there by suppresses the desired FM input.

When both are nearly equal strength, the receiver fluctuates back and forth between them.
phenomenon is known as Capture effect.

Radio Receivers

FM Threshold Effect:

The output signal to noise ratio of an FM receiver is given by

$$(SNR)_o = \frac{k_f^2 P}{\frac{2N_o w^3}{3A_c^2}} = \frac{3A_c^2 k_f^2 P}{2N_o w^3}$$

This is valid only if the carrier to noise ratio at the discriminator input is high compared with unity.

When the noise is increased, the carrier to noise ratio is decreased. At first, individual clicks are h
in the receiver output and as the carrier to noise ratio is decreases still further, the clicks rapidly m
into a crackling or sputtering sound.
Then onwards the receiver starts predicting larger values of SNR than the actual ones.
This phenomenon is called Threshold Effect.
The threshold effect is defined as minimum carrier to noise ratio giving an FM improvement wh
not significantly degraded from the value predicted by actual SNR.
For qualitative discussion of FM threshold effect, consider the signal x(t) at equivalent IF filter ou
when there is no signal present, So that carrier wave is unmodulated. i.e.

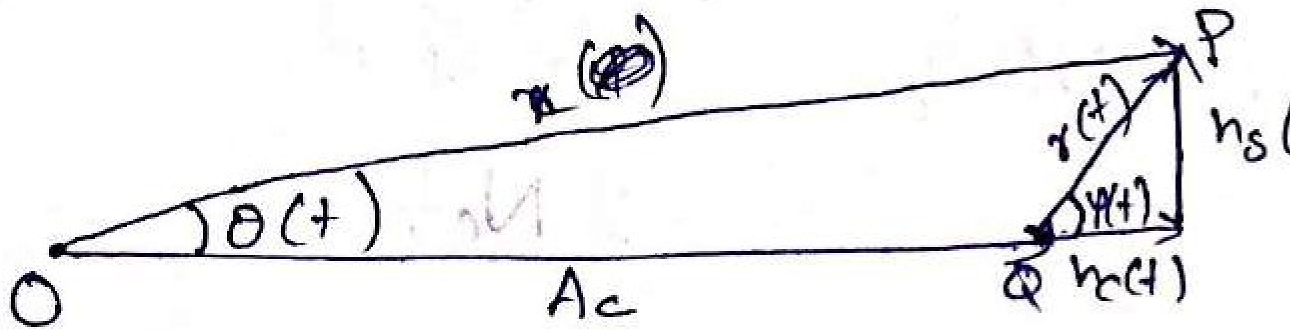
$$x(t) = [A_c + n_c(t)]\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t)$$

Radio Receivers

Where nc(t) and ns(t) are in phase and quadrature phase components of narrow band noise n(t).
In the above phasor diagram, r(t) is the envelope or magnitude of narrow band noise.

When carrier to noise ratio is large, $n_c(t)$ and $n_s(t)$ are usually much smaller than carrier amplitude A
So that Point P wanders near point.

When noise starts increasing, PQ crosses 'O'. Based on the direction of the phasor PQ either positive
negative clicks heard.

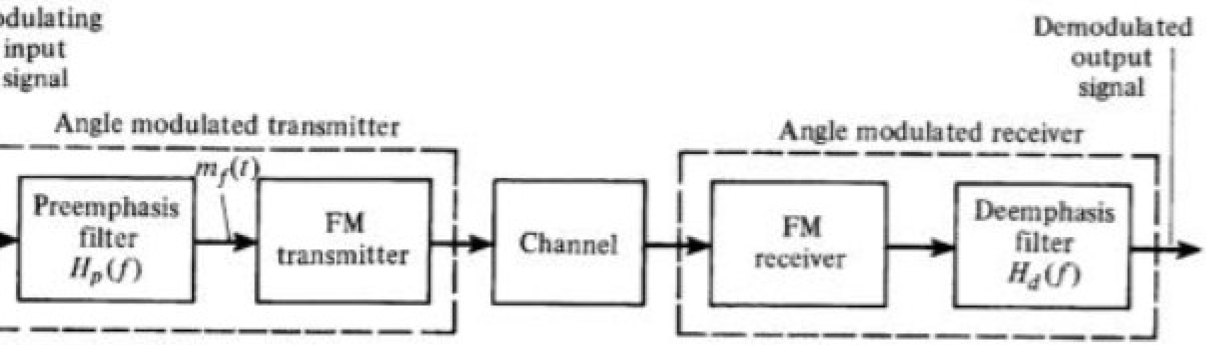


re-emphasis and De-emphasis.

- In FM, the noise increases linearly with frequency. By this, the higher frequency components of message signal are badly affected by the noise.
- To solve this problem, we can use a pre emphasis filter of transfer function Hp(f) at the transmitter to boost the higher frequency components before modulation.
- Similarly, at the receiver, the deemphasis filter of transfer function Hd(f) can be used after demodulator to attenuate the higher frequency components thereby restoring the original message signal

emphasis and De-emphasis.

e combined effect of pre-emphasis and de-emphasis is to increase high-frequency components during the transmission so that they l be stronger and not masked by noise



Overall Block Diagram

Pre-Emphasis:

Applies a high-pass filter to the signal before transmission

Boosts or amplifies the high-frequency compon

Commonly boosts frequencies above 2-3 kHz

Provides up to 10 dB of gain to high frequencies

Done prior to transmission or recording

Takes advantage of high-frequency noise immur

De-Emphasis:

Applies a low-pass filter to the received signal

Attenuates or reduces the boosted high frequencies

Rolls-off highs above 2-3 kHz

The reverse process of pre-emphasis

Provides gain reduction equal to the pre-empha

Restores original frequency spectrum

Reduces noise and distortion picked up during transmission

Pre-Emphasis

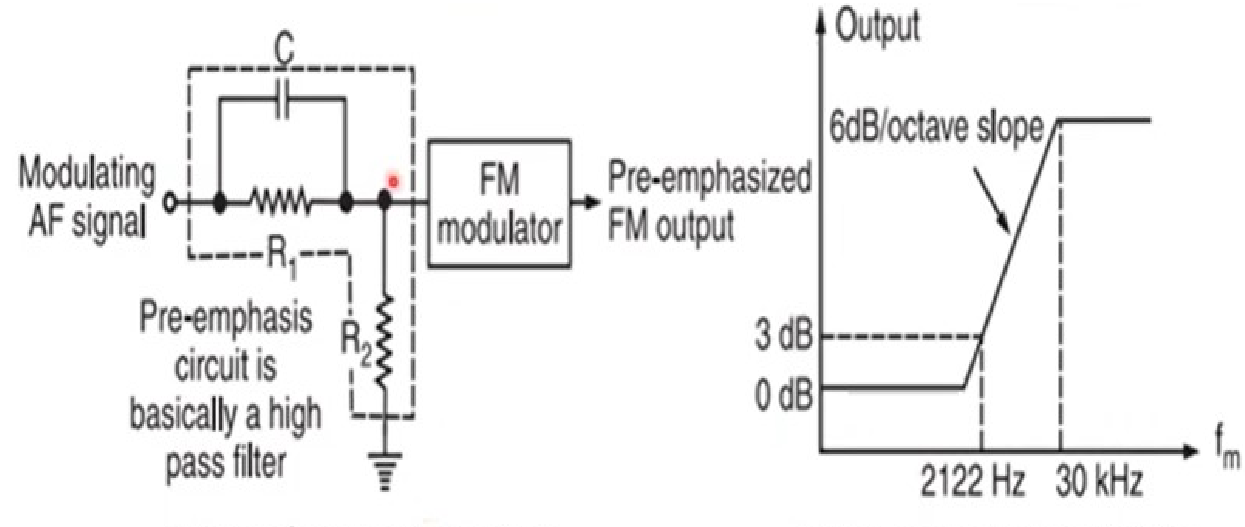
- Pre and de-emphasis circuits are used only in frequency modulation.
  - Pre-emphasis is used **at transmitter** and de-emphasis **at receiver**.

1. Pre-emphasis

- In FM, the noise has a greater effect on the higher modulating frequencies.
- This effect can be reduced by increasing the value of modulation index (m_f), for higher modulating frequencies.
- This can be done by increasing the deviation 'δ' and 'δ' can be increased by increasing the amplitude of modulating signal at higher frequencies.

Definition:

The artificial boosting of higher audio modulating frequencies in accordance with prearranged response curve is called pre-emphasis.



(a) Typical pre-emphasis circuit (b) Pre-emphasis characteristics

$$H(\omega) = \frac{R_2}{R_1+R_2}\,\frac{(1+sCR_1)}{(1+sC\,R_1 \| R_2)}$$

- As modulating frequency (f_m) increases, capac reactance decreases and modulating voltage goes on increa

  $f_m \propto$ Voltage of modulating signal applied to FM mod

  Boosting is done according to pre-arranged curve as sh in Fig. 2.

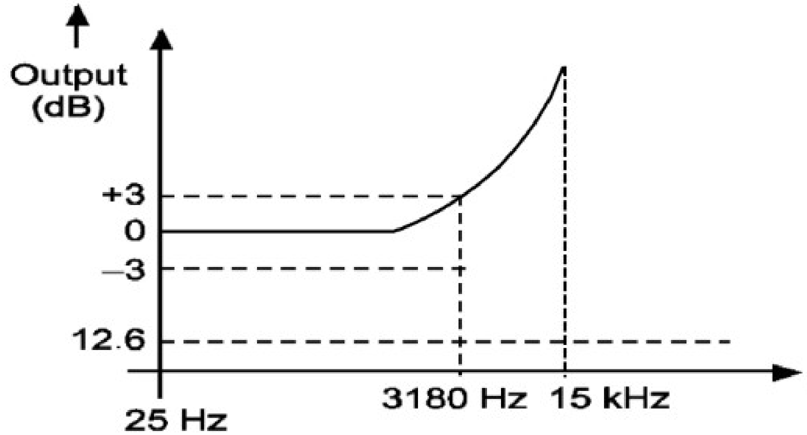


Fig. 2: P re-emphasis Curve

e-emphasis

- De-emphasis circuit is **used at FM receiver**.

finition:

The artificial boosting of higher modulating frequencies in th ocess of pre-emphasis is nullified at receiver by process called -emphasis.

- De-emphasis circuit is a low pass filter shown in Fig.

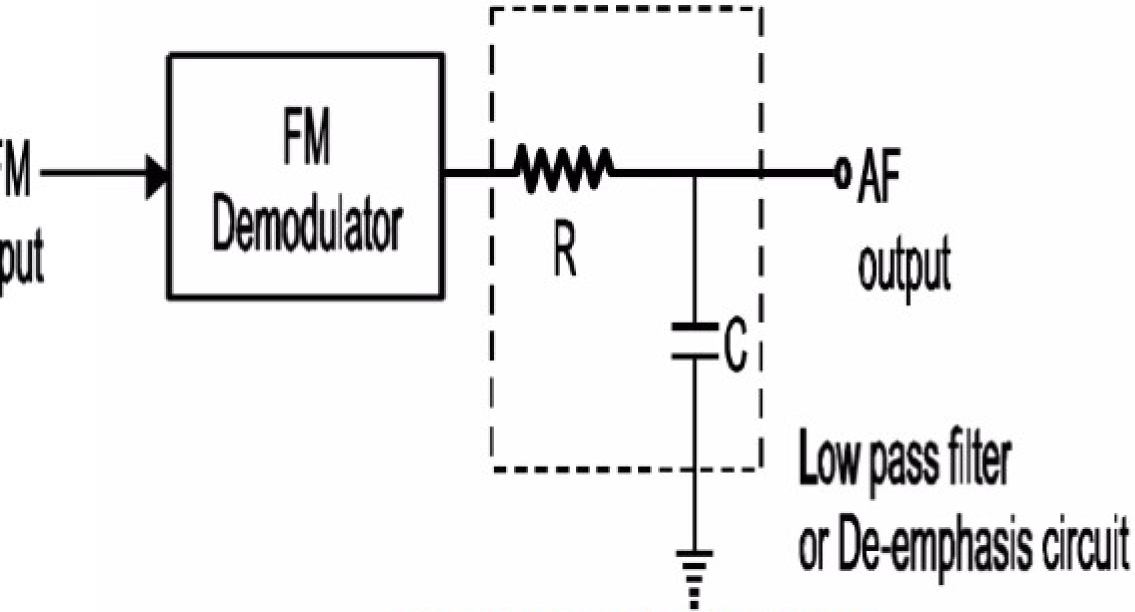
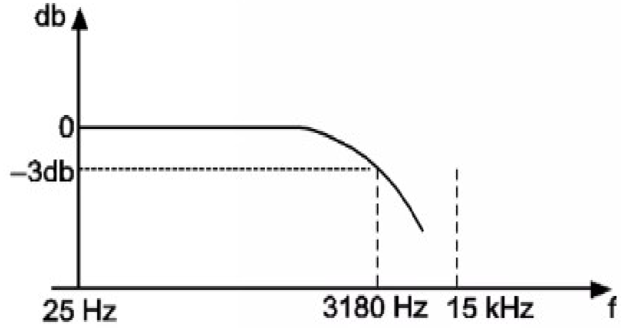


Fig. De-emphasis Circuit

$$H(f) = \frac{1}{1 + sRC}$$

Fig. De-emphasis Curve



As shown in Fig.5, de-modulated FM is applied to the de-emphasis circuit (low pass filter) where with increase in f_m, capacitive reactanc decreases. So that output of de-emphasis circuit also reduces •

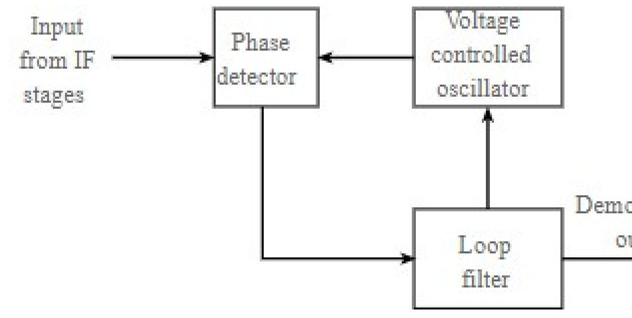
Fig. 5 shows the de-emphasis curve corresponding to a time constant

50 µs. A 50 µs de-emphasis corresponds to a frequency response cur that is 3 dB down at frequency given by,

$$f = 1/\,2\pi RC$$
$$= 1/\,2\pi \times 50 \times 1000$$
$$= 3180 \text{ Hz}$$

PLL FM Demodulators

The phase locked loop, PLL is a very useful RF building block. The PLL uses the concept of minimising the difference in phase between two signals: a reference signal and a local oscillator to replicate the reference signal frequency. Using this concept it is possible to use PLLs for many applications from frequency synthesizers to FM demodulators, and signal reconstitution.



PLL Phase locked Loop FM demodulator

To look at the operation of the PLL FM demodulator take the condition where no modulation is applied anc
carrier is in the centre position of the pass-band the voltage on the tune line to the VCO is set to the mid positio
However if the carrier deviates in frequency, the loop will try to keep the loop in lock. For this to happen the
frequency must follow the incoming signal, and in turn for this to occur the tune line voltage must vary.
Monitoring the tune line shows that the variations in voltage correspond to the modulation applied to the signa
amplifying the variations in voltage on the tune line it is possible to generate the demodulated signal.
Although no basic changes to the phase locked loop are required for it to be able to demodulate FM, a b
amplifier is typically provided from the tune line to prevent the tune line being loaded by other sections of
receiver. It provides a lower output impedance and as a result, this prevents loading from the audio amplifier
upsetting the loop in any way.